04/02/25

Probablity & Statistics

Statistics
- Classical
- Bayes

Variables
- Discrete
- Continuous

1) Nominal :- Based on nomenclature (No ranking, ordered & associative nature here)

2) Ordinal :- Only ordering occurs (rank).

3) Interval :- Interval scale [a, b] → like this.
   - Difference b/w absolute zero & zero.
   - No absolute zero. only zero. I consider both similar in nature

4) Radio scale :- If we need to 'comparison'.

Data :- collection of information with a specific objective.

- Primary — when you collect & you read
- Secondary — "when it is published".

It's easy to use sec data/dealing with sec data as it is already handled.

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Trial :- outcome is an event.

Exhaustive cases :- A set of events that must occur at least once during an experiment.

favourable cases :- The number of outcomes that result in the desired event.

Mutually exclusive :- Mutually exclusive events are events that can't occur at the same time.

Independent :- The probablity of an ~~even~~ event doesnot effect each other's probablity of happening.

Equally likely :- They are the events that have the same probablity of occurring.

Mathematical probablity :- (no of favourable case)

$P(A) = m/n$ → (It is only applicable to finite no. countable sets)

n → no. of exhausted case

$\lim_{n \to \infty} P(A) = \lim_{n \to \infty} m/n$ → empirical definition of probability

→ It is applicable until the variable is discrete

Axiom :- Predefined Principle Assumption :-

~~Exumatic~~

- Axiomatic approach :-

applicable Problem of continuity.

$f(A) =$ [redacted] $\Omega = \omega = S = \{ \Phi, H, T \}$.

Axiom :-

i) $P(A) \leq 1$ Non-negativity condition / positivity of probablity

ii) $P(S) = 1$ (Assurity of sure event)

iii) $P\left(\bigcup_{i=1}^{m} A_i\right) = \sum_{i=1}^{n} P(A_i)$ → applicable for independent event

(Additive property)

$P(A_1) \cdot P(A_2) \cdots P(A_n) = P(A_1) + P(A_2) + \cdots + P(A_n)$

$P(A) = 0$

if A is rare event.

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i) $0 \leq P(A) \leq 1$

ii) $P(S) = 1$

iii) $P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$ → Probablity law.

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ → Mutually exclusive → event not occeur simultaneously
- $P(A \cup B) = P(A) + P(B)$ → disjoint ↓ don't have any impact with each other.

code → 16x+3+z Google classroom

Conditional probability :- Let A & B are events (dependent)

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, \quad P(B/A) = \frac{P(B \cap A)}{P(A)}$$

↳ B already happened ; A already happened

multiplication law of probability

$$\boxed{P(A \cap B) = P(A/B) \cdot P(B)}$$

$\boxed{P(A \cap B) = P(A) \cdot P(B)}$ ← when event A is independent.

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot P(B)}{P(B)} = P(A)$$

- **Random variable :-** Real valued function from sample space S to real space (R)

  x, y, z ...

- **Probability Distribution :-**

  Distribution function → F(x).

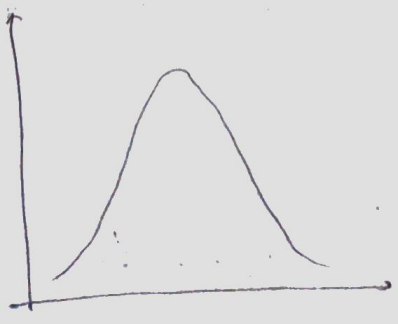
  Probability density / mass fun^n = f(x) / P(x)

If there is a random variable, there always exists its corresponding distribution function

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- **Probability Mass func^n :-** A discrete random variable X with the possible outcomes $x_1, x_2, \ldots, x_n$ corresponding having the probabilities $P(x_1), P(x_2), \ldots, P(x_m)$. If $P(x_i) > 0$ & $\sum_{i=1}^{n} P(x_i) = 1$ then the func^n $P(x_i)$ is known as probability mass function.

$\sum_{i=1}^{n} P(x_i)$ = cummulative distributed func^n



The sample space is given as $S = \{HH, HT, TH, TT\}$
Let the random variable of getting the no. of heads is X. so $X(s) = \{0, 1, 2\}$. Therefore the probability of getting 2 heads $P(x=2) = 1/4$, the probability of getting one head $P(x=1) = 2/4 = 1/2$ and no. head $= 1/4$

x	0	1	2
$P(x)$	$1/4$	$1/2$	$1/4$

$$P(X=x) = \sum P(x_i)$$
↳ for discrete

Property of cummulative distributed function

1) $P[a < x < b] = f(b) - f(a) - P[x=b]$
2) $P[a \leq x \leq b] = f(b) - f(a) - P[x=a]$
3) $P[a < x \leq b] = f(b) - f(a)$
4) $P[a \leq x \leq b] = f(b) - f(a) - P[x=b] + P[x=a]$

→ for continuous

- Mathematical Expectation :-

$$E(x) = \sum x_i P(x_i) \longrightarrow \text{for discrete} \quad \boxed{\mu = \bar{x}}$$

$$E(x) = \int_{x/R} x f(x)\, dx \longrightarrow \text{for continuous}$$

* Expectation is a mean.

~~Actual definition~~ →

x_i	2	3	4	5	6	7	8	9	10	11	12
$P(x_i)$	$1/36$	$2/36$	$3/36$	$4/36$	$5/36$	$6/36$	$5/36$	$4/36$	$3/36$	$2/36$	$1/36$

$$E(x) = \sum_{i=1}^{n} x_i P(x_i) = 2 \times 1/36 + 3 \times 2/36 + 4 \times 3/36 + \ldots + 12 \times 1/36 = 7.$$

Properties of mathematical exp.

1) $E(ax) = aE(x) = a\mu$
2) $E(k+x) = k + E(x)$
3) $E(ax \pm b) = aE(x) \pm b$

1) $E(ax) = \int ax f(x)\, dx = a \int x f(x)\, dx = aE(x)$
2) $E(k+x) = \int (k+x)\, dx = k \int f(x)\, dx + \int x f(x)\, dx$
$= k + E(x).$

3) $E(ax \pm b) = \int (ax \pm b) f(x)\,dx = a\int x f(x)\,dx \pm b f(x)\,dx$

$= aE(x) \pm b.$

- **Variance :-**

If the expectation $E(x) = 5$. then find $5 + 3x + 2x$.

Sol $y = 5 + 3x + 2x$

$E(y) = E(5 + 3x + 2x)$

$= 5 + 3E(x) + 2E(x)$.

$\frac{1}{N}\sum x_i$

$= 5 + 3 \times 5 + 2 \times 5$

$= 5 + 15 + 10 = 30$ (Ans)

- **Variance :-** $\boxed{\sigma = \sqrt{\frac{1}{N}\sum (x_i - \bar{x})^2}}$ → standard deviation

$\boxed{V(x) = \sigma^2 = \frac{1}{N}\sum (x_i - \bar{x})^2}$ → variation about mean

$V(x) = \sigma^2 = \frac{1}{N}\sum_{i=1}^{n} (x_i - A)^2$ → variation about any arbitrary value A

$\boxed{E(x) = \bar{x} = \frac{1}{N}\sum x_i}$

$\boxed{V(x) = E(x^2) - [E(x)]^2}$

- **Properties of variance :-**

i) $V(ax) = a^2 V(x)$

ii) $V(ax \pm b) = a^2 V(x)$

iii) $V(a + x) = V(x)$

Q) A variance has the following distribution :-

x	-3	6	9
$P(x)$	$1/6$	$1/2$	$1/3$

Calculate $E(x)$, $E(x^2)$, $E[(2x+1)^2]$

Sol) $E(x) = \sum_{i=1}^{3} x P(x_i)$

$= -3 \times 1/6 + 6 \times 1/2 + 9 \times 1/3$

$= -\frac{1}{2} + 3 + 3 = 5.5$

ii) $E(x^2) = \sum x^2 p(x_i)$

$= 9^3 \times 1/4 + 36 \times 1/2 + 81 \times 1/3$

$= 3/2^2 + 18 + 27$

$= 3/2 + 45 = \frac{3+90}{2} = \frac{93}{2} = 46.5$.

iii) $E\left[(2x+1)^2\right]$

$= E\left[4x^2 + 4x + 1\right]$

$= 4E(x^2) + 4E(x) + 1$.

$= 4 \times 46.5 + 4 \times 5.5 + 1$

$= 209$

iv) $V(x) = E(x^2) - [E(x)]^2$

$= 46.5 - (5.5)^2$

$= 46.5 - 30.25$

$= 16.25$

- **Moments** :- The nth moment of a random variable x is $\mu^n = \mu_n = E[x^n] = \begin{cases} \int_x x^n f(x)\,dx, & \text{if continuous} \\ \sum_x x^n p(x_i) & \text{if discrete} \end{cases}$

  → Raw moment

  If discrete, then, $\mu_n = \mu^n$

- **Central Moment** :-

  $\mu_n = E\left[(x - \bar{x})^n\right]$ → nth moment about mean / nth central moment.

  $= \begin{cases} \int_x (x-\bar{x})^n f(x)\,dx & \text{→ continuous} \\ \sum_x (x-\bar{x})^n p(x_i) & \text{→ discrete.} \end{cases}$

  $\mu_n = E\left[(x-A)^n\right]$ → about arbitrary n^{th} moment

  $= \begin{cases} \int_x (x-A)^n f(x)\,dx \\ \sum_x (x-A)^n p(x_i) \end{cases}$

- **Binomial distribution** :-

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- $\mu_0 = \mu_0' = 1$
  $\mu_1 = 0$
  $\mu_2 = \mu_2' - \mu_1'^2 = V(x) - \sigma^2$
  $\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2\mu_1'^3$
  $\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_1'^2\mu_2' - 4\mu_1'^4$

→ Relation b/w Raw moment & Central moment.

- **Moment Generating function :-**

$$M_x(t) = E[e^{tx}]$$

$$= \begin{cases} \sum_i e^{tx_i} p(x_i) & \longrightarrow \text{if } x \text{ is discrete} \\ \int_x e^{tx} f(x)\,dx & \longrightarrow \text{if } x \text{ is continuous} \end{cases}$$

$$E(x) = \begin{cases} \sum_i x_i p(x_i) \\ \int_x x f(x)\,dx \end{cases}$$

- **properties of Moment Generating function :-**

i) $M_{cx}(t) = M_x(ct)$

L.H.S → $M_{cx}(t) = E[e^{ctx}]$ $(\because M_x(t) = E[e^{tx}]$ — ①

R.H.S → $M_x(ct) = E[e^{ctx}]$ — ②

So from ① and ② L.H.S = R.H.S

ii) The moment generating func" of the sum of numbers of independent random variables is equal to the product of their respective moment generating function.

$$M_{x_1+x_2+x_3+\cdots+x_n}(t) = M_{x_1}(t) \cdot M_{x_2}(t) \cdot M_{x_3}(t) \cdots M_{x_n}(t).$$

proof :- $M_{x_1+x_2+\cdots+x_n}(t) = E[e^{(x_1+x_2+x_3+\cdots+x_n)t}]$

L.H.S -

$$= E[e^{x_1 t} \cdot e^{x_2 t} \cdot e^{x_3 t} \cdots e^{x_n t}]$$

$$= E[e^{x_1 t}] \cdot E[e^{x_2 t}] \cdots E[e^{x_n t}]$$

$$= M_{x_1}(t) \cdot M_{x_2}(t) \cdots M_{x_n}(t).$$

R.H.S = Hence proved.

iii) change of origin and scale of MGF :-

* If we multiply or divide in the coordinates then it is called the shift of scale.
* The phenomenon of adding or subtracting in the given coordinates then it is called the shift of origin.

Let us transform a random variable x to U which is defined as $U = \frac{x-a}{h}$ [$\because a$ = effect of change of origin, h = interval of spacing.

where a and h are constants.

Therefore the moment generating func^n of random variable U is given as $M_u(t) = E\left[e^{ut}\right] = E\left[e^{\left(\frac{x-a}{h}\right)t}\right]$

$$= E\left[e^{xt/h} \cdot e^{-\frac{at}{h}}\right]$$

$$= e^{-at/h} E\left[e^{xt/h}\right]$$

$\because t$ = time

$$= e^{-at/h} M_x(t/h)$$

iv) If the moment generating func^n of a distribution exists, then it is uniquely determined.

* It is not always necessary that MGF of a random variable exists. It may or may not exists.

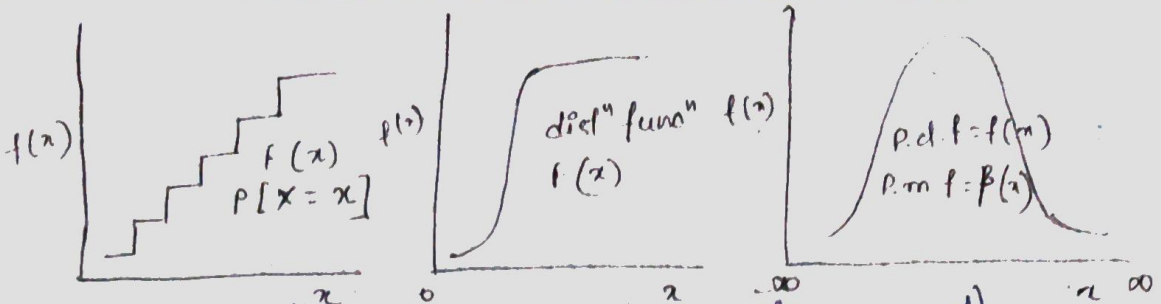
$$M_x(t) = E\left[e^{tx}\right]$$

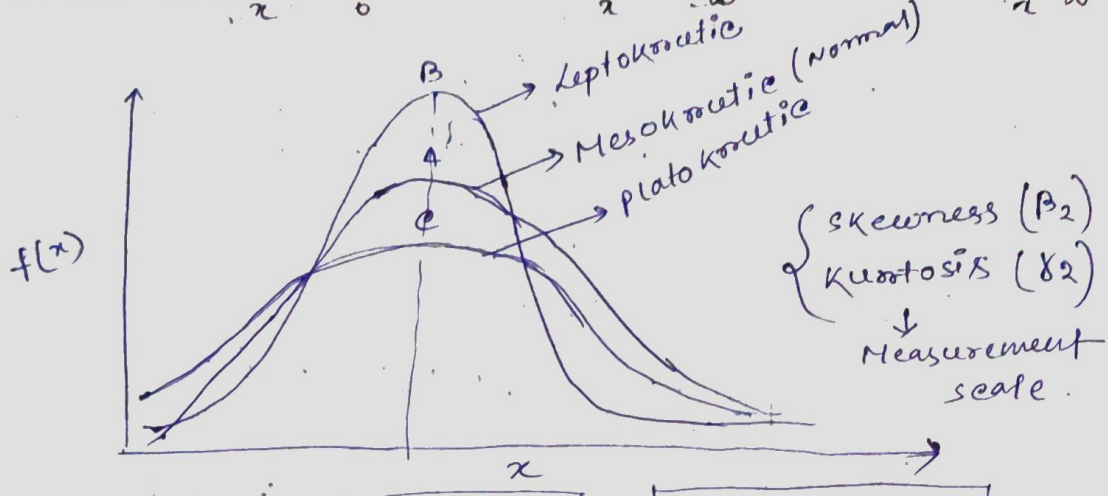
$$= E\left[1 + tx + \frac{(tx)^2}{2!} + \cdots\right]$$

$$= 1 + tE(x) + t^2\frac{E(x^2)}{2!} + t^3\frac{E(x^3)}{3!} + \cdots$$

$$= 1 + t\mu_1' + \frac{t^2}{2!}\mu_2' + \frac{t^3}{3!}\mu_3' + \cdots$$

$$= \boxed{\sum_{r=0}^{\infty} \frac{t^r}{r!}\mu_r' = M_x(t)}$$





$$\beta_1 = \frac{\mu_3^3}{\mu_2^3} \; ; \; \beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\gamma_1 = \sqrt{\beta_1}$$

$$\gamma_2 = \beta_2 - 3$$

for Mesokrutic = $\beta_2 = 3$; $\gamma_2 = 0$

for platokrutic = $\beta_2 < 3$; $\gamma_2 < 0$

for Leptokrutic = $\beta_2 > 3$; $\gamma_2 > 0$

- **Binomial Distribution**:- whose outcomes only 2.
  Generalisation of Bernoulli Distribution.
  Let X be a random variable having PMF $P(x)$; where $x = 0, 1, 2, \ldots, n$. Then the ~~probablity~~ PMF is defined as $P(\underset{=x}{X}) = {}^{n}C_x \, p^x \, q^{n-x}$; $x = 0, 1, 2, \ldots, n$ & $p + q = 1$
  where $\overset{n}{\alpha}$ and p are said to be parameter of binomial dist^n. & usually denoted as ~~$X \sim B(n, p)$~~ $X \sim B(n, p)$
  $n \longrightarrow$ no. of trials
  $p \longrightarrow$ probablity of successes.
  $p + q = 1$ [where q is prob of failure]

$X \sim B(n, p)$

$P(X = x) = {}^{n}C_x \, p^x q^{n-x}$; $x = 0, 1, 2, \ldots, n$

- **Physical condition for binomial distribution :-**

i) Each trial results in two exhaustive mutually exclusive & disjoints outcomes termed as success & failure.

ii) The no. of trials 'n' should be finite.

iii) The probablity of success 'p' is constant for each trial.

iv) The trials are independent of each other.

- **Moments of binomial distribution :-**

i) $\bar{x} = \mu_1' = E(x) = \sum_{x=0}^{n} x \cdot {}^{n}C_x \cdot p^x \cdot q^{n-x}$

$= np \sum_{x=1}^{n} {}^{n-1}C_{x-1} \, p^{x-1} q^{n-(x-1)}$

$\bar{x} = np$

$\boxed{\bar{x} = np}$

$B((n-1), p)$

$\sum_{x=0}^{n} {}^{n}C_x \, p^x \cdot q^{n-x}$

$= 1$

$\sum P(x_i) = 1$

ii) $\bar{x} = \mu_2' = E(x^2) = \sum_{x=0}^{n} x^2 \, {}^{n}C_x \cdot p^x \cdot q^{n-x}$

$= \sum \{ x(x-1) + x \} {}^{n}C_x \, p^x q^{n-x}$

$= n(n-1) p^2 \left\{ \sum_{x=2}^{n} {}^{n-2}C \cdot p^{x-2} q^{n-x} \right\} + np$

$= n(n-1) p^2 (p+q)^{n-2} + np$

$\mu_2' = E(x^2) = n(n-1)p^2 + np.$

$V(x) = E(x^2) - [E(x)]^2$

$= n(n-1)p^2 - n^2 p^2 + np$

$= (n^2 - n) p^2 - p^2 n^2 + np$

$\Rightarrow p^2 n^2 - np^2 - p^2 n^2 + np$

$= np(1-p)$ $\because (1 - p = q)$

$\therefore p + q = 1$

$\boxed{V(x) = npq}$

$E(x) > V(x)$ → for binomial distribution.

Standard deviation $(x) = \sqrt{\text{Var}(x)}$

$SD(x) = \sqrt{npq}$

- ~~Additive prop~~
- Moment generating func" of Binomial distribution:-

M.G.F $\Rightarrow X \sim B(n,p)$:-

$$M_x(t) = E[e^{tx}]$$

$$= \sum_{x=0}^{n} e^{tx} \cdot {}^nC_x \cdot p^x \cdot q^{n-x}$$

$$= \sum_{x=0}^{n} {}^nC_x (e^t p)^x q^{n-x}$$

$$= M_x(t) = (e^t p + q)^n$$

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- Additive properties of Binomial distribution :- [$x_1 + x_2 \sim B(n_1 + n_2)$]

Let $x_1 \sim B(n_1, p_1)$ & $B(n_2, p_2)$

$M_{x_1}(t) = (p_1 e^t + q_1)^{n_1}$

& $M_{x_2}(t) = (p_2 e^t + q_2)^{n_2}$

Then we have to show that $(x_1 + x_2)$ follows Binomial dist" with parameters $(n_1 + n_2)$ and $(p_1 + p_2)$

$$x_1 + x_2 \sim B(n_1 + n_2, p_1 + p_2)$$

$$M_{x_1 + x_2}(t) = M_{x_1}(t) \cdot M_{x_2}(t)$$

$$= (p_1 e^t + q_1)^{n_1} \cdot (p_2 e^t + q_2)^{n_2}$$

- Remark :- Here for the given probablity p_1 & p_2 moment generating func" x_1 & x_2 $M_{x_1+x_2}(t)$ is not expressive form of $(pe^t + q)^n$ then form in general Binomial distribution does not follow additive property. However the property probablity of success i.e., p_1 & p_2 are equal in such case $p_1 = p_2 = p$, $M_{x_1+x_2}(t) = (pe^t + q)^{n_1} (pe^t + q)^{n_2}$

$= (pe^t + q)^{n_1 + n_2}$ such that $x_1 + x_2 \sim B(n_1 + n_2, p)$

- Poisson distribution :- A random variable x is said to follow poison distribution if its probablity mass func" (p.m.f) is defined as follows $P[x = x] = \frac{e^{-\lambda} \lambda^x}{x!}$, $x = 0, 1, 2, \ldots$ $\lambda > 0$ (λ is paramder of poison)

it can be denoted as follows $x \sim P(\lambda)$

Example :- The number of misprints on a page in a book. The no. of peoples living in a community age 100 years. The no. of wrong telephone number dialed in a day decay of alpha particles count

• Moment Generating function poisson distribution :-

$$M_x(t) = E(e^{tx}) = \sum_{x=0}^{\infty} e^{tx} \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= e^{\lambda} \sum_{x=0}^{\infty} \frac{e^{tx} \lambda^x}{x!}$$

$$= e^{-\lambda} e^{t} \sum_{x=0}^{\infty} \frac{e^{xt} \lambda^x}{x!}$$

$$= e^{-\lambda} \left[1 + \lambda e^t + \frac{(\lambda e^t)^2}{2!} + \cdots \right]$$

$$= e^{-\lambda} \cdot e^{\lambda e^t} \quad \left(\text{Because } e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)$$

$$= M_{xt} = e^{\lambda(e^t - 1)}$$

• Rth Moment :- $\mu'_n = E[x^n]$

$$= \sum_{x=0}^{\infty} \frac{x e^{-x} \lambda^x}{x!}$$

$$= \boxed{e^{-\lambda} \sum_{x=0}^{\infty} \frac{x^n \lambda^x}{x!}}$$

• Rth Moment :- $\mu'_n = E[x^n]$

$$= \sum_{x=0}^{\infty} \frac{x^n e^{-\lambda} \lambda^x}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{x^n \lambda^x}{x!}$$

Substituting $n = 1$

$$\mu_1' = e^{\lambda} \sum_{x=0}^{\infty} \frac{x \lambda^x}{x!}$$

$$= \lambda e^{-\lambda} \left[1 + \lambda + \frac{\lambda^2}{2!} + \cdots\right]$$

$$= \lambda e^{-\lambda} \cdot e^{\lambda} = \lambda$$

$$\therefore \boxed{E(x) = \bar{x} = \lambda}$$

$$E_1(x_1^2) = \sum_{x=0}^{\infty} x^2 \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= \lambda^2 + \lambda$$

$$V(x) = E(x)^2 - [E(x)]^2$$

$$\Rightarrow \boxed{V(x) = \lambda}$$

$$\Rightarrow V(x) = \lambda^2 + \lambda - (\lambda)^2$$

Substituting $n = 2$

$$\mu_2' = e^{-\lambda} \sum_{x=0}^{\infty} \frac{x^2 \lambda^x}{x!}$$

$$= \sum_{x=0}^{\infty} \frac{x^2 e^{-\lambda} \lambda^x}{x!}$$

$$= \lambda^2 + \lambda$$

Additive properties of poisson's distribution:

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If x_i, $i = 1, 2, \ldots n$ follows poisson's distribution with parameter λ_i, $i = 1, 2, \ldots n$ then $\sum x_i$'s are said to follow additive property.

If $\sum x_i$ s follow poisson's distribution with parameter $\sum \lambda_i$

i.e., $x_i \sim P(\lambda_i) \Rightarrow \sum x_i \sim P(\sum \lambda_i)$

$M_x(t) = e^{\lambda(e^t - 1)}$

$\Rightarrow M_{x_1 + x_2 + x_3 + \cdots + x_n}(t) = M_{x_1}(t) \cdot M_{x_2}(t) \cdots M_{x_n}(t)$

$= e^{\lambda_1(e^t - 1)} \cdot e^{\lambda_2(e^t - 1)}$

$$\boxed{M_{\sum x_i}(t) = e^{\sum \lambda_i (e^t - 1)}}$$

Normal Distribution :- A random variable x is said to follow Normal distribution with parameter μ (mean) & σ^2 (variance) if its probability density func^n is given as $\longrightarrow$

$$f(x, \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2\sigma^2}(x-\mu)^2\right\} \quad ; -\infty < x < \infty \; ; -\infty < \mu < \infty \; ; \sigma^2 > 0$$

Let $z = \frac{x - \mu}{\sigma}$ (change of origin & scale)

$$\Rightarrow \boxed{f(z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{z^2}{2}\right\}} \quad ; -\infty < z < \infty$$

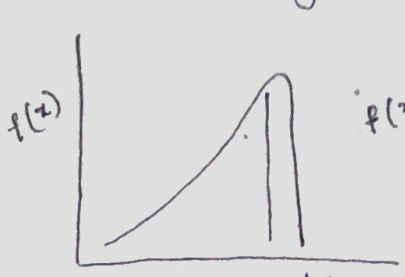
with mean zero & variance 1

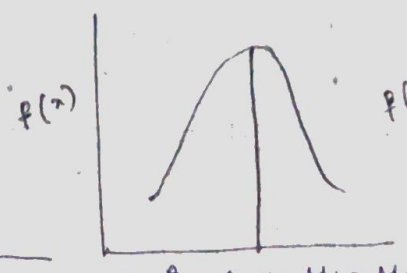
Pdf of standard Normal variable/Dist^n

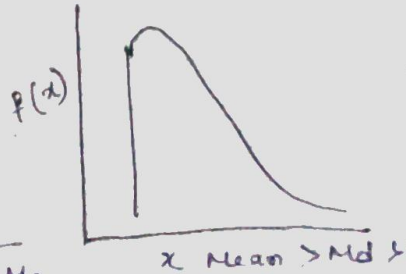
* Normal distribution is also known as symmetric distribution

$$\boxed{\phi(z) = \phi(-z)}$$

* Normal dist^n also follows symmetry.
* " " graph is also called Bell-Graph.







$$\boxed{f(-x) = f(x)}$$

* Normal dist^n can be considered as a limiting case of binomial dist^n under the condition.

i) $n \to \infty$

ii) p & q are not approaching to zero.

p & $q \not\to 0$

~~iii) Normal~~

* Normal dist^n can be considered as a limiting case of poisson's dist^n under the situation →

i) ~~$n \to \infty$~~

ii) ~~p & q~~ i) $\lambda = \infty$

iii) ~~$\lambda \to \infty$~~

* Binomial dist^n can be considered as a limiting case of poison's dist^n under the condition.

i) $n \to \infty$

ii) p & $q \to 0$.

- If $x \sim N(15, 10)$, what is the Pdf value of the func^n

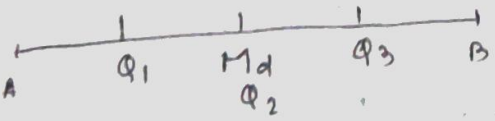
$\mu = 15$, $\sigma^2 = 10$ $\sigma = \sqrt{10}$.

- Moments :-

The odd central moments of a normal dist^n are always zero.

i.e., $\boxed{\mu_1 = 0, \mu_2 = \sigma^2, \mu_3 = 0, \mu_4 = 3\sigma^4}$

$\boxed{Q = \text{Quantile}}$



For normal dist^n → $\boxed{Md - Q_1 = Q_3 - Md}$

- Quatile Deviation :- $\boxed{Q.D = \frac{Q_3 - Q_1}{2}}$

* For normal dist^n $Q.D = \frac{Q_3 - Q_1}{2} = \frac{2}{3}$

- Mean Deviation :- $M.D \approx \frac{4}{5}\sigma$

- Standard deviation :- $S.D = \sqrt{\text{var}(x)}$

$Q.D : M.D : S.D = 2/3\sigma : 4/5\sigma : \sigma = 10 : 12 : 15.$

- **Median of Normal dist^n** :- for a dist^n, median is a pt. which divides the dist^n into 2 equal parts.

$$\int_{-\infty}^{M} f(x)dx = 1/2 \quad \text{so} \quad \int_{-\infty}^{M} f(x)\,dx = \int_{M}^{\infty} f(x)\,dx = 1/2$$

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{M} \exp\left\{-\frac{1}{2}\left(\frac{x-u}{\sigma}\right)^2\right\} du = 1/2$$

$$\frac{1}{\sigma\sqrt{2\pi}} \left[\int_{-\infty}^{M} \exp\left\{-1/2\left(\frac{x-u}{\sigma}\right)^2 du + \int_{u}^{M} \exp\left\{-1/2\left(\frac{x-u}{\sigma}\right)^2 du\right]\right] = 1/2$$

Let $z = \left(\frac{x-u}{\sigma}\right)$

$\Rightarrow \sigma\,dz = dx$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{-\frac{1}{2}z^2} dz + \int_{M}^{M} f(x)\,dx = 1/2$$

$$\Rightarrow 1/2 + \int_{u}^{M} f(x)\,dx = 1/2$$

$$\int_{u}^{M} f(x)\,dx = 0$$

since $f(x)$ can't be zero: this implies $\longrightarrow$

$$\boxed{M = u}$$

- **Mode of Normal dist^n** :- The mode of the dist^n is the value of x for which $f(x)$ is maximum.

Here the dis^n under consideration is normal dist^n therefore $\rightarrow$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-1/2\left(\frac{x-u}{\sigma}\right)^2\right\} \quad \text{— (1)}$$

Mode is the sol^n of $f'(x) = 0$ & $f''(x) < 0$.

Taking log on both sides of eq^n 1

$$\log f(x) = \left\{\log \frac{1}{\sigma\sqrt{2\pi}} - \frac{1}{2}\left(\frac{x-u}{\sigma}\right)^2\right\} \log e$$

Differentiating it w.r.t x.

$$f'(x) = 0 \quad \text{(2)}$$

$$\Rightarrow f'(x) = -\left(\frac{x-u}{\sigma^2}\right) f(x) \longrightarrow \therefore \boxed{x = u}$$

• for max Again differentiating

$$f''(x)\Big|_{\mu} = -\left[f'(x)\left(\frac{x-\mu}{\sigma^2}\right)\Big|_{x=\mu} - \frac{1}{\sigma^2} f(x)\Big|_{x=\mu}\right]$$

$$= -\frac{f(\mu)}{\sigma^2} \Rightarrow \boxed{x = \mu}$$

• Moment Generating func" for Normal dist" :-

$$M_x(t) = E[e^{tx}]$$

$$= \int_{-\infty}^{\infty} e^{tx} f(x)\, dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{tx} e^{\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}} dx$$

Let $z = \frac{x-\mu}{\sigma}$

$\Rightarrow \sigma\, dz = dx$

$$\Rightarrow \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t(\sigma z + \mu)} e^{-1/2 z^2} dz$$

$$= \frac{e^{t\mu}}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left\{-1/2\left(z^2 - 2t\sigma z\right)\right\} dz$$

$$= \frac{e^{t\mu}}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left[-1/2\left(z - \sigma t\right)^2 - \sigma^2 t^2\right] dz$$

$$= e^{\mu t + 1/2\sigma^2 t^2} \times \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-1/2 u^2} du$$

$$\boxed{M_x(t) = e^{\mu t + 1/2\sigma^2 t^2}}$$

• Additive properties of Normal dist" :-

$X_1 \sim N(\mu_1, \sigma_1^2)$

$X_2 \sim N(\mu_2, \sigma_2^2)$

$X_1 \pm X_2 \sim N(\mu_1 \pm \mu_2, \sigma_1^2 + \sigma_2^2)$

22/04/25

• Bivariant / Two dimensional Random Variable :-

Joint probablity Mass function (J.P.M.F) :- Let (x, y) be two random variables in a space $x = S_1 = \{x_1, x_2, x_3 \ldots x_n\}$ and $y = S_2 = \{y_1, y_2, y_3 \ldots y_n\}$ then the joint spaces $= x(s) \times y(s) = \{x_1, x_2, \ldots, x_m\} \times \{y_1, y_2, \ldots, y_n\}$. The function P on the joint space can be defined as in the following table.

X \ Y	$y_1, y_2, \ldots y_n$	Total
x_1	$P_{11}\ P_{12} \ldots P_{1n}$	$P_1.$
x_2	$P_{21}\ P_{22} \ldots P_{2n}$	$P_2.$
⋮	⋮	⋮
x_m	$P_{m1}\ P_{m2} \ldots P_{mn}$	$P_m.$
Total	$P_{.1}\ P_{.2} \ldots P_{.n}$	1

$$\sum_{i=1}^{n}\sum_{j=1}^{m} P_{ij} \qquad \boxed{\sum P_{i.} = \sum P_{.j} = 1}$$

$$\sum_{j=1}^{m} P_{.i}\ ,\ \sum_{i=1}^{n} P_{i.}$$

• If (x, y) is the two dimensional discrete random variable then the joint ~~density~~ mass function denoted by P_{xy} is defined as $P_{xy}(x_i, y_j) = P(X = x_i, Y = y_j) \forall (x, y) \in (x_i, y_j)$, 0 otherwise.

Marginal Probablity function :-

Let (x, y) be two dimensional random variable then the probability distribution of x is defined as $P(x) = P(X = x_i)$

$$= P(X = x_i \cap Y = y_1) + P(X = x_i \cap Y = y_2) \ldots P(X = x_i \cap Y = y_j)$$

$$P(X) = P_{i1} + P_{i2} + \ldots + P_{im} = P_{i.}$$

$$P(Y) = P_{1j} + P_{2j} + \ldots + P_{mj} = P_{.j}$$

$x = 0, 1, 2$

$y = 1, 2, 4$

$P_{xy}(x = 2, y = 5) = 0$

Q) The following table represents the joint probability Mass of the given variable (x, y)

$x \backslash y$	1	2	Total
1	0.1	0.2	0.3
2	0.1	0.3	0.4
3	0.2	0.1	0.3
Total	0.4	0.6	1

- find the marginal distribution of x & y, the conditional distribution of x given $y = 1$. find.
Find the probablity of $x+y$, $P[(x+y) < 4]$.

Ans i) the marginal of x

$P_x(X = x)$

x	1	2	3
$P(x)$	0.3	0.4	0.3

$P_Y(Y = y)$

y	1	2
$P(y)$	0.4	0.6

→ Marginal of Y

ii) $P(X=1 \mid Y=1) = \dfrac{P(X=1 \cap y=1)}{P(y=1)}$

$= \dfrac{0.1}{0.4} = 1/4$

$P(X=2 \mid y=1) = \dfrac{P(x=2 \cap y=1)}{P(y=1)}$

$= \dfrac{0.1}{0.4} = 1/4$

$P(X=3 \mid y=1) = \dfrac{0.2}{0.4} = 1/2$

iii) $P[(x+y) < 4] = P[X=1, Y=1] + P[X=2, y=1] + P[X=1, y=2]$

$= 0.1 + 0.1 + 0.2 = 0.4$

- **Joint Density function :-** for a random variable (x, y) the joint probability density func" is defined as $f_{xy}(x,y) = \frac{\partial^2 F(x,y)}{\partial x \partial y}$

$$= \lim_{\substack{\delta x \to 0 \\ \delta y \to 0}} \frac{P(x < X < x+\delta x,\ y < Y < y+\delta y)}{\delta x\, \delta y}$$

- **Marginal Density function :-** $f_x(x) = \frac{d f_{xy}(x,y)}{dx} = \int_{-\infty}^{\infty} f(x,y)\,dy$

$$f_y(y) = \frac{d f_{xy}(x,y)}{dy} = \int f(x,y)\,dx$$

↳ marginal of y.

marginal of x.

- **Independence of Random Variable :-** Two random variable (x,y) is said to be independent if $f_{xy}(x,y) = f_x(x) \cdot f_y(y)$

$$f_{x_1, x_2, \ldots x_n}(x_1, x_2, \ldots x_n) = f(x_1) \cdot f(x_2) \cdots f(x_n) = \prod_{i=1}^{n} f(x_i)$$

I.I.D → Independent Identity Distributed.

$X_1 \sim E(\theta)$; $f(x_1) = \theta e^{-\theta x_1}$ → Exponential.

$f(x_2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2}\left(\frac{x_2 - \mu}{\sigma}\right)^2\right]$ ↳ Normal

$-\infty < x < \infty$
$-\infty < \mu < \infty$
$0 < \sigma^2 < 0$

$f_{x_1 x_2}(x_1, x_2) = f(x_1) \cdot f(x_2)$.

Let x_1 & x_2 are i.i.d $\sim N(\mu, \sigma^2)$

Q) Let x & y are two continuous random variable & their J.P.D.F is given as $f(x,y) = \begin{cases} 6x^2 y & ; 0 < (x+y) < 1 \\ 0 & ; \text{otherwise} \end{cases}$

i) varifying ~~the given Joint P.D~~ it is a joint P.D.F.

ii) find $P[0 < x < 3/4,\ 1/3 < y < 2]$, $P[x + y < 1]$,

$P[x > 4]$ & $P[x < 1 \mid y < 2]$

Soln i) $\int_x \int_y f(x,y) = 1$ [for J.P.D.F]

$\int_0^1 \int_0^1 6x^2 y\,dx\,dy$.

$$\int_0^1 \int_0^1 6x^2 y \, dx \, dy$$

$$= \int_0^1 6 \left[y \cdot \frac{x^3}{3} \right]_0^1 dy$$

$$\Rightarrow \int_0^1 y \cdot 1/3 \, dy = 6/3 \left[\frac{y^2}{2} \right]_0^1$$

$\Rightarrow 2 \times 1/2 = 1$ so it is a J.P.D.F

ii) $P[0 < x < 3/4, 1/3 < y < 2]$

$$= \int_0^{3/4} \int_{1/3}^{2} 6x^2 y \, dx \, dy$$

$$= 6 \left[\quad \right]_0^{3/4} x^2 y \, dx \, dy + \int_{1/2}^{2} x^2 y \, dy$$ → it will be 0 as it is not defined.

$$= 6y \left[\frac{x^3}{3} \right]_0^{3/4} = 6 \cdot y \cdot \frac{(3/4)^3}{3} = 2y \cdot \frac{27}{64}$$

$$= \frac{27}{32} y$$

- **Regression Analysis** :- study of average relationship b/w 2 or more random variable is known as regression.

↓

study of one or two or more variable is known as Regression.

* statistical model takes account of associated error

- **Regression Analysis** :- study of average relationship b/w 2 or more random variable is known as regression.

↓

study of one or two or more variable is known as Regression.

* statistical Model takes account of associated errors

06|05|25

$$cov(x,y) = E\left[\{x - E(x)\}\, y - E(y)\}\right]$$

$$= \frac{1}{n}\sum_{i=1}^{n} xy - \bar{x}\bar{y} \Rightarrow E(xy) = - E(x)E(y) = 0$$

&

ii) $cov(ax+by, cx+dy) = ac\,cov(x,y) + bd\,cov(y,x)$

iii) $cov(x+a, y+b) = cov(x,y)$

iv) $cov\left(\sum_{i=1}^{n} a_i x_i, \sum_{j=1}^{m} b_j y_j\right) = \sum_{i=1}^{n}\sum_{j=1}^{m} a_i b_j\, cov(x_i, y_j)$

Correlation coefficient: $r = \dfrac{cov(x,y)}{\sigma_x \sigma_y}$ [Ratio, index, indices.

- Theorem:- Show that correlation coefficient is independent of change of origin and scale. Prove.

→ Let x & y be two random variables.

Let us assume a transformation $u = \dfrac{x-a}{h}$ & $v = \dfrac{y-b}{k}$ — (1)

$\Rightarrow x = uh + a$ & $y = vk + b$

Taking expectation,

$E(x) = E[a + uh]$

$= a + hE(v)$ — (2)

$E(y) = b + kE(v)$ — (3)

$V(x) = \sigma_x^2 = V(a + uh)$

$= h^2 V(u)$ — (4)

$V(y) = \sigma_y^2 = V(b + vk)$

$\Rightarrow \sigma_y^2 = k^2 V(v)$ — (5)

Now $r(x,y) = \dfrac{cov(x,y)}{\sigma_x \sigma_y}$ ——— (*)

$cov(x,y) = E[\{x - E(x)\}\{y - E(y)\}]$

$= E[h\{u - E(u)\}k\{v - E(v)\}]$

$= hkE[\{u - E(u)\}\{v - E(v)\}]$

$= hk\,cov(u,v)$ — (6)

$\rightarrow r(x,y) = \dfrac{hk\,cov(u,v)}{hk\,\sigma_x \sigma_y} = \dfrac{cov(u,v)}{\sigma_x \sigma_y} = r(u,v)$

— substitute the value from (4) (5) (6) we get

→ r only measures linear relationship. $y = x^2$

→ $r = \pm 1$ (range) ($+1$ → perfect positive correlation.
-1 → " negative "
0 → No linear relation)

$y = a + b + c$

Regression line y on $x = y - \bar{y} = b_{yx}(x - \bar{x})$: $b_{yx} = \frac{\sigma_{xy}}{\sigma_x^2}$

b_{yx} is known as coefficient regression.

$(x - \bar{x}) = b_{xy}(y - \bar{y})$

$b_{xy} = \frac{\sigma_{xy}}{\sigma_y}$

- Angle b/w line of regression :-

$$\tan\theta = \frac{|m_1 - m_2|}{1 + m_1 m_2} = \frac{\sigma_x \cdot \sigma_y}{\sigma_x + \sigma_y} \left| \frac{1 - r^2}{r} \right|$$

$\sigma_x = \sigma_y = \sigma$

$\tan^{-1}\left(\frac{4}{3}\right)$ Find r ?

→ $\sigma_x = \sigma_y = \sigma$

$\theta = \tan^{-1}\left(\frac{4}{3}\right) = \frac{\sigma\sigma}{2\sigma\sigma}$

$= \frac{\sigma\sigma}{2\sigma}\left(\frac{1 - r^2}{r}\right)$

$\frac{4}{3} = \frac{\sigma\sigma}{2\sigma}\left(\frac{1 - r^2}{r}\right)$

→ Smarus

- Hypothesis :- Null hypothesis is a statement made for carrying out a test, it is tested for its possible rejection.

E.g → Ho : $\mu = 90\%$

→ The statement is stated against the null hypothesis is known as alternative hypothesis. $\mu \neq 90\%$.

→ The when

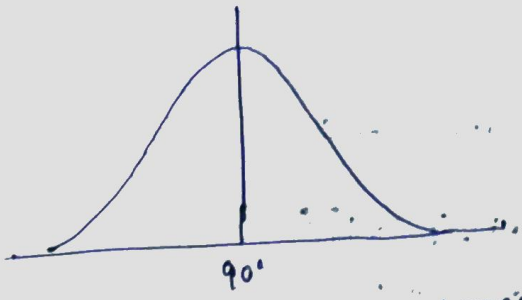
i) A hypothesis is completely specified is known as simple hypothesis. E.g $H_1 : \mu = 50\%$

→ when a hypothesis is not completely specified is known as composite hypothesis.

Eg i) $H_1 : \mu \neq 90\%$.

ii) $H_{20} : \mu < 90\%$. → R. Tail test

iii) $H_0 : \mu > 90\%$. → L. Tail Test.



→ • chance causes • Assignable causes.

• Types of errors :-

1) Rejecting null hypothesis when it is true.
2) Accepting null hypothesis when it is false.

Level of significance :- / size of test - $\alpha = P$ (reject ; when true).

$\beta = P$ (accept ; when false).

Power of test $= 1 - \beta$

Criteria of goodestimator

$E(t) - \theta = 0$

(i) Unbiased
(ii) Consistent / consistency
(iii) Efficiency
(iv) Sufficiency

Let $x_1, x_2, \ldots, x_n$ is a random sample from normal population $\sim N(\mu, 1)$. Show that $t = \frac{1}{n}\sum_{i=1}^{n} x_i^2$ is an unbiased estimator of $\mu^2 + 1$.

we are given that $E(x_i) = \mu$

$V(x_i) = 1$

5 marks

$\Rightarrow V(x) = E(x^2) - [E(x)]^2$

$E(x^2) = V(x) + E(x)^2$

$= 1 + \mu^2$

$E(t) = \frac{1}{n} E\left(\sum_{i=1}^{n} x_i^2\right)$

$= \frac{1}{n}\left[n(1 + \mu^2)\right]$

$\Rightarrow E(t) = 1 + \mu^2$. prove.

* If T an unbiased estimator of θ. show that T^2 is biased estimator of θ^2.

→ since T is unbiased estimator of θ

$\Rightarrow E(T) = \theta$.

We know that $V(T) = E(T^2) - [E(T)]^2$

$\Rightarrow V(T) = E(T^2) - \theta^2$

$\Rightarrow E(T^2) = V(T) + \theta^2$

since $V(T) > 0$, $E(T^2) \neq V(T) + \theta^2$